Sensitivity of Total Strain Energy of a Vehicle Structure to Local Joint Stiffness

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This paper discusses how the total strain energy of a vehicle structure is affected by local joint stiffness. By using the principle of virtual work and the minimum strain energy theorem, a closed-form expression for the sensitivity coefficient has been derived. The insensitivity of the total strain energy of a vehicle structure to a particular joint, when its stiffness exceeds a certain value (or threshold value), has been proved mathematically. In order to investigate the sensitivity of the structure to the joint stiffness, a "stick" model has been created and its results have been compared to the test data for accuracy. Some data on joint stiffness of tested vehicles are also presented.

Nomenclature

b_m = mth parameter
D_{jk} = compliance matrix
J_m = mth joint stiffness
n_j = vector normal to the boundary surface S, shown in Fig. 3
N = number of joints
N_b = total number of parameters
N_r = total number of redundancies
p_i = generalized force vector
q_j = generalized strain vector
Q_j = generalized stress vector
Q_j^w = free component of Q_j
Q^r = reactant component of Q_j
S = surface of the structure
S_p = surface where the force vector p_i is prescribed
S_u = surface where the displacement vector u_i is prescribed
u_i = generalized displacement vector
U = total strain energy
U_m = strain energy stored in the volume V_m due to the external loads p_i $(i=1,...,N_p)$
U_s = total strain energy stored in a "stick" model
U_{sm} = strain energy stored in the mth joint under given external loads
V_m = volume in which D_{jk} depends on b_m
α_p = joint stiffness multiplication factor

Introduction

OVER the years, the study of the joint behavior of vehicle structures has been identified as one of the most important subjects in the automotive industry. It is widely known that the flexibility of structural joints can affect not only the noise, vibration, and harshness characteristics of a vehicle, but also other vital structural performance characteristics under various loading conditions (e.g., crash loads, road loads, jacking load, towing load, etc.).

The first study which accounted for the effect of flexible joints on automotive structural responses was by Chang[1] who used a two-dimensional frame model for a static analysis. He found that the structural response is far more sensitive to reducing joint stiffness (relative to the baseline values) than to increasing them. Recently a similar phenomenon was reported by Du and Chon,[2] and it was claimed that there might exist a threshold stiffness value in a joint of a given vehicle structure. In other words, if a joint stiffness exceeds the threshold value, then the total strain energy of the structure becomes *insensitive* to the particular joint.

The present paper demonstrates the above phenomenon theoretically by showing that the derivative of the total strain energy with respect to a particular joint stiffness decreases and becomes zero as the joint stiffness approaches infinity. In this paper, a closed-form expression for the sensitivity coefficient has been derived using the principle of minimum strain energy and the principle of virtual work. In order to investigate the sensitivity of the structure to joint stiffness, a "stick" model (see Fig. 1) has been created according to the concept described in Ref. 2. (A stick model is a beam-only finite-element model of a vehicle structure; however, unlike the beam model, the sheet metal panels are not included and the joints are considered as perfect.) This modeling concept is based on the assumption that beams/frames are the primary load-carrying members in a structure. In order to investigate the accuracy of the model, the overall deformed shapes obtained from the analyses and the tests for both bending and torsion are compared in Figs. 2a and 2b. The dotted and solid lines represent the test data and the analysis results, respectively. The abscissa denotes length of the body structure from the front to the rear wheel axles and, thus, represents the length of the wheel base. The ordinates denote deflections for the bending analysis and twist angles for the torsional analysis. The overall deformed shape from the analysis is in good agreement with that of the test.

The last section discusses joint behavior and compares the analytical results with test data. Discussion of other components is also given based on the sensitivity coefficients derived in the paper.

Basic Concepts

This section summarizes the basic concepts of the general sensitivity study reported in Refs. 3–5. These will then be applied to joint behavior in the later section.

Theoretical Background

Linearity of the equilibrium and strain-displacement relations will permit the principle of virtual work to be written as:

$$\int_V Q_j q_j^* \, dV = \int_S p_i u_i^* \, dS \qquad (1)$$

where p_i and Q_j are any statically admissible fields, and u_i^* and q_j^* are any kinematically admissible fields. In the present

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paper, the body forces are assumed to be negligible. Note that $S = S_p + S_u$ (Fig. 3).

Let the solution of a structural problem for an elastic material be given by u_i, Q_j, and q_j. These quantities constitute, by definition, both a statically admissible field and a kinematically admissible field. In addition, q_j and Q_k satisfy Hooke's law:

$$q_j = D_{jk} Q_k \tag{2}$$

Note that if the deformations are small, the total strain energy stored in the loaded system will be equal to the work done by the applied forces. Thus the total strain energy U may be expressed in terms of generalized stresses as

$$U = 1/2 \int_V D_{jk} Q_j Q_k \, \mathrm{d}V \tag{3}$$

Since a structure is, in general, statically indeterminate, one may divide the generalized stress $Q_j(x_\beta)$ at any point x_β into two parts:

$$Q_j(x_\beta) = Q_j^w(x_\beta) + \sum_{r=1}^{N_r} \lambda_j^r(x_\beta) Q^r \tag{4}$$

where $\lambda_j^r(x_\beta)$ $(r = 1, \ldots, N_r)$ are independent of the applied loads and depend only on the geometrical configuration of the structure.[6] Then, substituting Eq. (4) into Eq. (3) and using the principle of virtual work [Eq. (1)], one can prove that

$$\frac{\partial U}{\partial Q^r} = \int_V D_{jk} \lambda_j^r Q_k \, \mathrm{d}V = 0 \tag{5}$$

This is Castigliano's Theorem of Compatibility, often referred to as the Principle of Minimum Strain Energy.[6] Equation (5) implies that the quantity U is minimized with respect to the values of each of the redundancies; Eq. (5) thus yields exactly N_r equations from which the values of the redundancies may be found.

Sensitivity Analysis

The objective is to derive a closed-form expression for the sensitivity coefficient $\partial U / \partial b_m$. Differentiating the total strain energy U, which is defined in Eq. (3), with respect to the mth variable b_m, leads to the following expression:

$$\frac{\partial U}{\partial b_m} = \frac{1}{2} \int_S D_{jk} Q_j Q_k \frac{\partial x_\beta}{\partial b_m} n_\beta \, \mathrm{d}S + \frac{1}{2} \int_V \frac{\partial D_{jk}}{\partial b_m} Q_j Q_k \, \mathrm{d}V + \int_V D_{jk} \frac{\partial Q_j}{\partial b_m} Q_k \, \mathrm{d}V \tag{6}$$

Here Eq. (6) may be considered as material derivative of volume integral.[7] Equation (6) can be greatly simplified if one chooses certain types of parameters. For example, an appropriate choice of beam or plate/shell properties (e.g., material property, area, moment of inertia, etc.), will make the first term of Eq. (6) identical to zero. And since the free components Q_j^w in Eq. (4) are the solutions of the statically determinate structures, they are independent of beam or plate/shell properties. This allows a differentiation of Eq. (4) with respect to b_m into:

$$\frac{\partial Q_j}{\partial b_m} = \sum_{r=1}^{N_r} \lambda_j^r \frac{\partial Q^r}{\partial b_m} \tag{7}$$

Then, using the Minimum Strain Energy Principle [Eq. (5)] and Eq. (7), it can be shown that the last term of Eq. (6) also vanishes. Finally, one can rewrite Eq. (6) as

$$\frac{\partial U}{\partial b_m} = \frac{1}{2} \int_{V_m} \frac{\partial D_{jk}}{\partial b_m} Q_j Q_k \, \mathrm{d}V \tag{8}$$

It should be noted that the integration in Eq. (8) need only be performed over the region V_m in which D_{jk} depends on b_m.

In addition, if one can express the compliance tensor D_{jk} as inversely proportional to b_m (i.e., $D_{jk} \propto 1/b_m$) in the region V_m, then Eq. (8) can be further simplified:

$$\frac{\partial U}{\partial b_m} = -\frac{1}{b_m} \left(\frac{1}{2} \int_{V_m} D_{jk} Q_j Q_k \, \mathrm{d}V \right) = -\frac{U_m}{b_m} \tag{9}$$

Sensitivity Study of Joints

Thus far, the basic concept of the derivation of the sensitivity coefficients has been presented. The present section describes the application of the above results to the sensitivity study of joints which affect the total strain energy of a vehicle structure. As mentioned above, it has been analytically and experimentally demonstrated[1,2] that the joint behavior is one of the most important factors for the overall stiffness of the body structure. Note that the total strain energy is directly related to the overall stiffness of a structure. (The structure that contains higher strain energy is less stiff than the structure with lower strain energy under the same loading and boundary conditions.) For the sake of clarity, this section is divided into two subsections: the case involving a single joint and the case involving multiple joints.

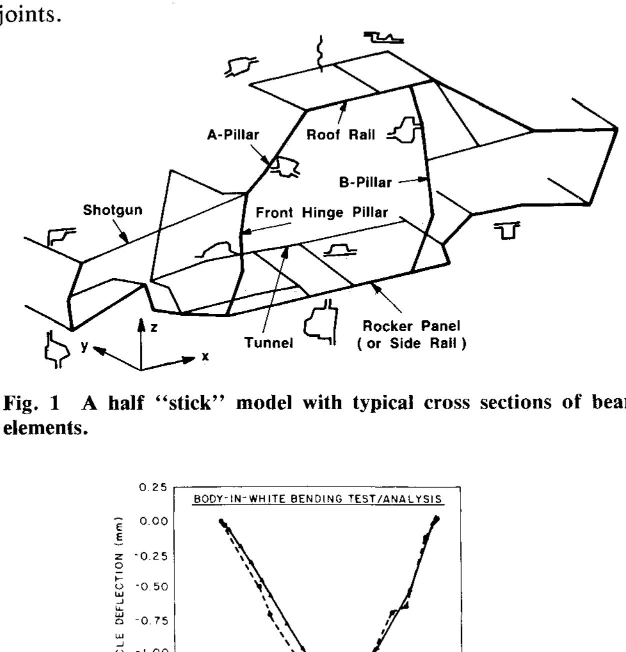


Fig. 1 A half "stick" model with typical cross sections of beam elements.

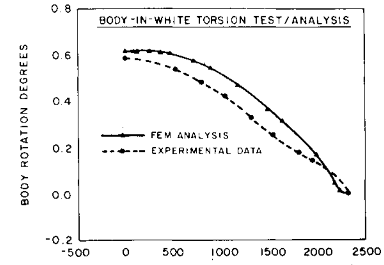


Fig. 2 Deflections vs length of a "stick" model for a) bending and b) torsion.

Single Joint

In the model analyzed, the joint which connects the rocker panel and the bottom of the B-pillar (see Figs. 1 and 4) was identified as the joint to which the total strain energy was most sensitive. This was done by comparing the amount of strain energy stored in the joints. After introducing a joint magnification factor, which was used in Ref. 2, a parametric study of the joint behavior was performed. Figure 5 shows how the total strain energy of the structure is affected by the joint stiffness of the B-pillar and the rocker panels. Note that the total strain energy becomes insensitive as the joint stiffness becomes large. This phenomenon can be explained using the sensitivity coefficient derived in the previous section [see Eq. (9)] as follows:

Let $b_m = J_m$ (the joint stiffness is a rotational spring constant opposed to the member stiffness which consists of the area, moments of inertia, and torsional constant for a given section) and let U_s be the total strain energy stored in the model under the prescribed loading conditions (either bending or torsion). Then Eq. (8) can be rewritten as

$$\frac{\partial U_s}{\partial J_m} = \frac{1}{2}\int_{V_m} \frac{\partial D_{jk}}{\partial J_m} Q_j Q_k \,\mathrm{d}V \tag{10}$$

Note that integration in Eq. (10) needs only be performed over the volume in which the mth joint is contained. Moreover, since the compliance tensor D_{jk} is inversely proportional to the mth joint stiffness, J_m, the final form of Eq. (10) is

$$\frac{\partial U_s}{\partial J_m} = -\frac{U_{sm}}{J_m} \tag{11}$$

It is very important to note from Eq. (11) that the sensitivity coefficient $\partial U_s/\partial J_m$ goes to zero as the mth joint stiffness, J_m, approaches infinity. Mathematically one can write this as

$$\lim_{J_m\to\infty} \frac{\partial U_s}{\partial J_m} = \lim_{J_m\to\infty}\left(-\frac{U_{sm}}{J_m}\right) = 0 \tag{12}$$

Equation (12) proves the phenomenon shown in Fig. 5 for large value of J_m (see region C). In addition, it should be noted that the total strain energy also becomes insensitive to J_m as it approaches zero (see region A in Fig. 5). This will be discussed in the next section.

Multiple Joints

Equation (12) can be generalized to compute a derivative of the strain energy with respect to more than one joint stiffness. Given a group of joints which are of interest, the associated joint stiffness multiplier α_p is defined as

$$(J_p,\ldots\ldots,J_{p+N}) = \alpha_p(\chi_p,\ldots\ldots,\chi_{p+N}) \tag{13}$$

The number of joints, N, in one group can be completely arbitrary. Then Eq. (10) can be modified as

$$\frac{\partial U_s}{\partial \alpha_p} = \sum_{\xi=p}^{N}\left(\frac{1}{2}\int_{V_\xi} \frac{\partial D_{jk}}{\partial \alpha_p} Q_j Q_k \mathrm{d}V\right) \tag{14}$$

Again, since $D_{jk} \propto 1/\alpha_p$, Eq. (14) becomes

$$\frac{\partial U_s}{\partial \alpha_p} = -\frac{1}{\alpha_p}\sum_{\xi=p}^{N}\left(\frac{1}{2}\int_{V_\xi} D_{jk} Q_j Q_k \mathrm{d}V\right) = -\frac{1}{\alpha_p}\sum_{\xi=p}^{N} U_\xi \tag{15}$$

Note that the individual strain energy has to be summed in this case. Therefore one can conclude that the following ex-

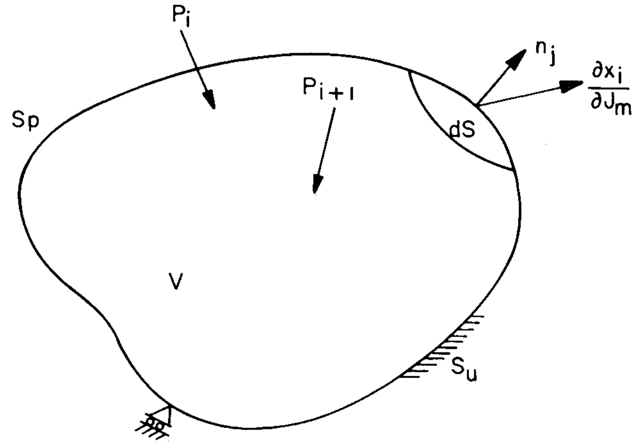
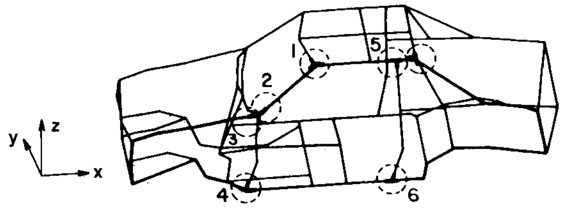


Fig. 3 A general body surface S with two parts, S_p and S_u.



1) A-Pillar to Roof Rail
2) A-Pillar to Front Hinge Pillar
3) Shotgun to Front Hinge Pillar
4) Front Hinge Pillar to Rocker
5) B-Pillar to Roof Rail
6) B-Pillar to Rocker

Fig. 4 Joint locations.

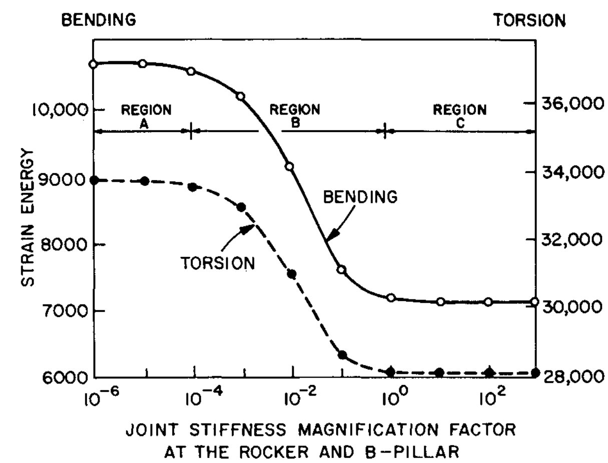


Fig. 5 Strain energy vs stiffness of the joint between the rocker panels and the B pillar.

Table 1 Measured joint stiffness values (see Fig. 4)

	Stiffness ($\times 10^7$ N-mm/rad)		
Joints	Vehicle A[a]	Vehicle B[a]	Vehicle C[a]
1	2.12/1.61	3.96/3.48	5.12/3.38
2	3.55/2.46	2.45/3.69	3.48/2.84
3	14.4/3.92	28.7/15.6	18.0/5.14
4	20.1/3.26	39.3/4.51	27.4/4.12
5	2.35/0.18	2.75/0.12	7.41/0.20
6	10.1/0.54	22.6/1.25	16.9/1.29

[a] Fore-aft/in-outboard values, respectively.

pression is also true:

$$\lim_{\alpha_p \to \infty} \frac{\partial U_s}{\partial \alpha_p} = 0 \tag{16}$$

In addition, Eq. (15) implies that the strain energy U_s is a *hyperbolic* function of the multiplication factor of the joint stiffnesses. Figure 6 shows the total strain energy variation as a function of the multiplication factor, α_p. Again, the total strain energy becomes far less sensitive if α_p exceeds a certain value. The present section offers explanation for the findings reported in Refs. 1 and 2.

Discussion and Conclusion

When stiffness of a single joint is varied, the total strain energy of a vehicle structure becomes sensitive to the local joint stiffness only within a certain stiffness range (region B, Fig. 5). In other words, the structure looses sensitivity not only when the joint stiffness is small (region A), but also when the joint stiffness is large (region C). The latter case has been proven in the previous section. For an explanation of the former case, one may consider the concept of a "failure mechanism" which has been used extensively in the literature on limit analysis.[8] Since the structure can sustain the given load with one or more "yield hinges," as long as the structure does not form a "mechanism," the structure can be said to have a finite stiffness, which is shown in region A of Fig. 5. This means that, even if one removes the particular joint, the structure will still sustain a load within given limits.

In the case of multiple joints, flexible joints have been introduced by adding 24 rotational spring elements at 12 structural joints in the "stick" model. The joints added in this fashion are shown in Fig. 4. A joint stiffness magnification factor [see α_p in Eq. (13)] was introduced, and a parametric study of the joint behavior was performed. Figure 6 shows a diagram of the total strain energy of the "stick" model vs the joint stiffness magnification factor for both bending and torsional loading cases. Published values for the joint stiffness obtained from three vehicle tests[9] (see Table 1) were used in the analyses. Table 2, as well as Fig. 6, compares the strain energy of the "stick" model (with perfect joints[2]) with the strain energy of the same model computed using the three sets of joint stiffness given in Table 1. It is interesting to note that the strain energy values using the three sets of joint stiffness are all within a range of 3% and that those values, compared with the values of the "stick" model differ by a maximum of 11%. This means that the actual values of joint stiffness may be equal to or slightly smaller than the corresponding threshold values. Unlike in the case of a single joint, the total strain energy becomes infinitely large as the multiplication factor approaches zero; this indicates that the joints shown in Fig. 4 may form a "failure mechanism."

These findings of the joints support the following: 1) A structure consisting of thin panels surrounded by frames, as is typical of automotive structures, may not be stiffened substantially by the panels under usual loading conditions, for the panels will buckle or deform like thin membranes, offering no support at the interior points. Even under these conditions, however, that part of the panel adjacent to the edge remains relatively undeformed, and acts as a gusset which stiffens the joint. This, then, implies that the joints can be treated as rigid in the model, reflecting the fact that the panels act as gussets; this, in turn, allows the joint stiffness to exceed the threshold value, and since the panels contribute negligibly to the stiffness of the structure away from the joints, they do not have to be explicitly included in the

Table 2 Comparison of strain energies

Strain energy	Bending	Torsion
U (Stick model)	7.04×10^3 N-mm (1.00)	2.88×10^4 N-mm (1.00)
U (Vehicle A)	7.79×10^3 N-mm (1.11)	3.17×10^4 N-mm (1.10)
U (Vehicle B)	7.69×10^3 N-mm (1.09)	3.07×10^4 N-mm (1.07)
U (Vehicle C)	7.57×10^3 N-mm (1.08)	3.11×10^4 N-mm (1.08)

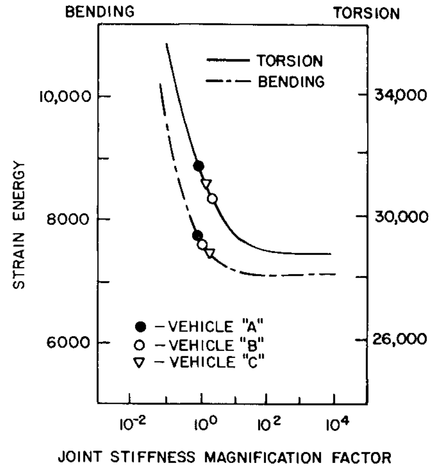


Fig. 6 Strain energy vs joint stiffness magnification factor.

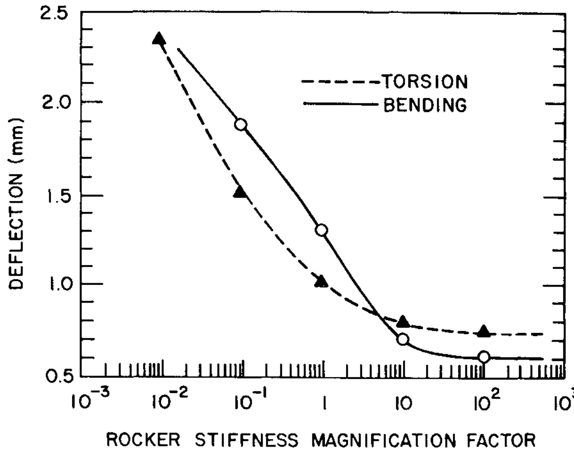


Fig. 7a Maximum deflections vs rocker stiffness magnification factor.

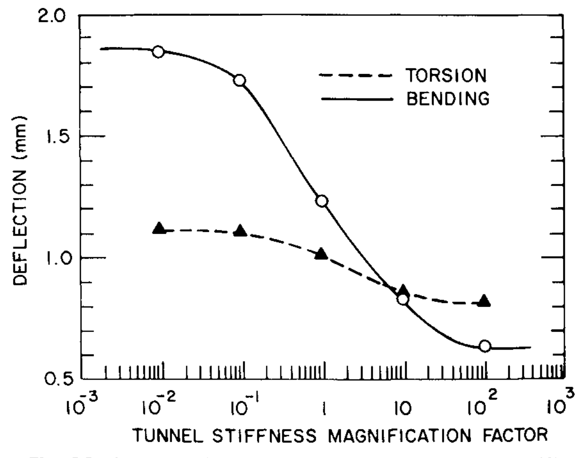


Fig. 7b Maximum deflections vs tunnel stiffness magnification factor.

model. 2) Specific quantitative design guidelines (threshold values) can be established for efficient joint construction of a vehicle structure. 3) A systems approach should be used for sensitivity study of a vehicle structure due to joint stiffness.

Finally, this idea which has been applied to the joints, can be extended to other components. For similar phenomena, it can be shown by means of varying stiffness values of components other than the joints. Figures 7a and 7b depict how the overall bending stiffness (solid lines) and torsional stiffness (dotted lines) change with the stiffness of the rocker panels or the tunnel. Figures 7a and 7b were generated by varying the stiffness (abscissa) of the rocker panels and the tunnel, respectively. The ordinates represent the maximum deflections for bending and the twist angles for torsion, respectively. It is obvious from both Figs. 7a and 7b that the overall vehicle stiffness is much more sensitive to the rocker panel than to the tunnel under bending as well as torsional loadings. One can, however, see that the curves of both figures become flat as the stiffness of these two components increases. This phenomenon can also be explained using the equations derived in the previous section by replacing the variable b_m with the stiffness of either the rocker panels or the tunnel.

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